## Pseudo Random Number Generation

Three Cases Where PRNGs Broke The System

```
>_ DEV v1.3-RC1
```

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E Cryptanalysis VO SS23
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ls.ecomaikgolf.com/slides/randomnumbers/

Motivation
(3) Why Random Number Generation

- Importance might be forgotten, we usually depend on them.
- We try to break the mode or the primitive, but not the RNG.
- Bad RNGs can take down cryptosystems.
(O) Objectives
- We wanted to show real world cases where RNGs broke the system
- For each case, explain the inner workings of the RNG and how they failed
- Plus a very special RNG ()


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- Naivest Case of bad RNG
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B A Novel Related Nonce Attack for ECDSA
Very Recent Attack
[0] 9.400.000 Dollars Affected
道 Dual Elliptic Curve Deterministic Random Bit Generator
IIII Standarized by NIST, ANSI, ISO for $7^{+}$Years
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## Elliptic Curve Essentials

(c) Which direction of computation is easy? (for known G)

$$
k \times G \rightleftharpoons P
$$

$$
k \in \mathbb{N}, \quad G, P \in \mathrm{EC}
$$

## Elliptic Curve Essentials

(P) Which direction of computation is easy? (for known G)

## $k \times G \stackrel{\text { easy }}{\rightleftharpoons} P$ hard

$$
k \in \mathbb{N}, \quad G, P \in E C
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Playstation 3
Nonce Misuse $\boldsymbol{\epsilon}$

## Introduction

ๆ Sony used Elliptic Curve Digital Signatures $\boldsymbol{a}$. for signed PS3 $\boldsymbol{\infty}$ software updates.

```
* ECDSA Recap
An ECDSA signature (r,s) can be created from a message m }\square\mathrm{ and a private key d Q.
2. We agree on:
    - A Fllintic Curve EC
    - Order n of G
    - A basis point G on EC
    - A hash function h
Algorithm:
    K& [1,n-1] Randomly choose from uniform distribution.
    R=kG=(x
    r=\mp@subsup{x}{R}{}\operatorname{mod}n If r=0 restart the algorithm.
    e=h(m)
    s=\mp@subsup{k}{}{-1}(e+d\timesr)\operatorname{mod}n\quad\mathrm{ If s=0 restart the algorithm.}
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An ECDSA signature $(r, s)$ can be created from a message $m \boxtimes$ and a private key d $\boldsymbol{Q}_{\text {. }}$
2. We agree on:

- A Elliptic Curve EC
- A basis point $G$ on EC

Algorithm:

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\begin{aligned}
k & \stackrel{\Phi}{\leftarrow}[1, n-1] \\
R & =k G=\left(x_{R}, y_{R}\right) \\
r & =x_{R} \bmod n \\
e & =h(m) \\
s & =k^{-1}(e+d \times r) \bmod n
\end{aligned}
$$

If $r=0$ restart the algorithm.

$$
\text { If } s=0 \text { restart the algorithm. }
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If $s=0$ restart the algorithm.

## Importance of Randomness

$$
s=\frac{e+d \cdot r}{k} \bmod n
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(?) What if an attacker gets to know $k$ ?

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(c) What if an attacker gets to know $k$ ?
(1) Private Key Q Recovery!

$$
s=\frac{e+d \cdot r}{k} \quad \longrightarrow \quad d=\frac{s \cdot k-e}{r} \bmod n
$$

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(:) Sony used the worst possible randomness: constant value $k$

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\begin{aligned}
& \text { int getRandomNumber() } \\
& \text { return 4; } / \text { // chosen by fair dice roll. } \\
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Source: xkcd 221
0. Discovered by group failOverflow (Dec. 2010) Key Recovery
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\frac{s_{1} \cdot k-e_{1}}{r_{1}} & =d=\frac{s_{2} \cdot k-e_{2}}{r_{2}} \\
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d & =\frac{s_{1}}{r_{1}} \cdot \frac{e_{1} \cdot r_{2}-e_{2} \cdot r_{1}}{s_{1} \cdot r_{2}-s_{2} \cdot r_{1}}-\frac{e_{1}}{r_{1}} \bmod n
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A Novel Related Nonce Attack for ECDSA

B

## Key observation

14 Recall $d=\frac{k_{i} s_{i}-h\left(m_{i}\right)}{r_{i}} \bmod n$.


- If these nonces obey a multivariate polynomial equation

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a_{0} k_{0}^{e_{0}}+a_{1} k_{1}^{a_{1}}+a_{2} k_{2}^{\theta_{2}}+\cdots+a_{N}=0
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( Furthermore, if $a_{i}$ and $e_{i}$ are known, the only unknown variable is $d$


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\frac{k_{0} s_{0}-h_{0}}{r_{0}}=\frac{k_{1} s_{1}-h_{1}}{r_{1}} \Longrightarrow k_{1}=\frac{r_{1} s_{0}}{r_{0} s_{1}} k_{0}+\frac{h_{1} r_{0}-h_{0} r_{1}}{r_{0} s_{1}}=u k_{0}+v
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## Recurrence relation PRNGs

Attack works if PRNG used to generate nonces:
$\square$ Uses arbitrary-degree recurrence relations modulo $n \rightarrow$ Only $k_{0}$ is truly random

(©) Goal
Produce a polynomial which only depends on the nonces, and not on unknown coefficients a;

## Recurrence relation PRNGs

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## Recurrence relation PRNGs

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k_{2} & =a_{N-3} k_{1}^{N-3}+a_{N-4} k_{1}^{N-4}+\cdots+a_{1} k_{1}+a_{0} \\
k_{3} & =a_{N-3} k_{2}^{N-3}+a_{N-4} k_{2}^{N-4}+\cdots+a_{1} k_{2}+a_{0} \\
\vdots & \\
k_{N-1} & =a_{N-3} k_{N-2}^{N-3}+a_{N-4} k_{N-2}^{N-4}+\cdots+a_{1} k_{N-2}+a_{0}
\end{aligned}
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Produce a polynomial which only depends on the nonces, and not on unknown coefficients $a_{i}$

## Example with Linear Congruential Generator PRNG

$$
\begin{aligned}
& k_{0} \stackrel{\$}{\leftarrow}[1, n-1] \\
& k_{1}=a_{1} k_{0}+a_{0} \\
& k_{2}=a_{1} k_{1}+a_{0} \\
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$$
\begin{aligned}
k_{1}-k_{2} & =a_{1}\left(k_{0}-k_{1}\right) \\
a_{1} & =\frac{k_{1}-k_{2}}{k_{0}-k_{1}} \\
k_{2}-k_{3} & =a_{1}\left(k_{1}-k_{2}\right) \\
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k_{2}= & a_{1} k_{1}+a_{0} & k_{2}-k_{3} & =a_{1}\left(k_{1}-k_{2}\right) \\
k_{3}= & a_{1} k_{2}+a_{0} & a_{1} & =\frac{k_{2}-k_{3}}{k_{1}-k_{2}} \\
& \left(k_{1}-k_{2}\right)^{2}-\left(k_{2}-k_{3}\right)\left(k_{0}-k_{1}\right)=0 \Longleftarrow \frac{k_{1}-k_{2}}{k_{0}-k_{1}}=\frac{k_{2}-k_{3}}{k_{1}-k_{2}}
\end{array}
$$

## Impact

Q. Private keys from vulnerable signature sets can be found quickly.
(1) Under 1 s for a small number of related nonces $N$
(ㄱ) $\sim 6.5 \mathrm{~s}$ for $N=16$, which yields a 92-degree polynomial
$\mathbf{\#}$ The Bitcoin blockchain was tested (for $\mathrm{N}=5$ )
l.all 424 million unique nublic kevs
$\rightleftarrows 9.1$ million unique public keys with at least 5 signatures $\square$

- 762 unique bitcoin wallets broken!

勫 All of them reused nonces and had zero balance. (2)
\$ Before they were exploited, these wallets contained about 144 BTC ( $\sim 9.4 \mathrm{M}$ USD)Ethereum blockchain was also tested
(8) No nractical success
(1) Many unexplored applications remain, since ECDSA is widely used.

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\$ Before they were exploited, these wallets contained about 144 BTC ( $\sim 9.4 \mathrm{M}$ USD)
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(1) Many unexplored applications remain, since ECDSA is widely used

## Impact

Q. Private keys from vulnerable signature sets can be found quickly.
© Under 1s for a small number of related nonces $N$
(1) $\sim 6.5$ s for $N=16$, which yields a 92-degree polynomial
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|lll 424 million unique public keys
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Dual Elliptic Curve
Deterministic Random
Bit Generator

## Introduction

II DUAL＿EC＿DRBG was a cryptographically secure deterministic random bit generator

```
f* History
- Developed by the NSA 有 along others such as HASH_DRBG
- Originally standarized by ANSI, NIST IIII and ISO followed
- Available in NIST's SP 800-90A 昷 (10.6028/NIST. SP. 800-90Ar1)
- Deprecated from SP 800-90A in 2014 (from 2006)
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- Makes use of Elliptic Curve Cryptography © (Cryptography VO L8)
- Uses two Elliptic Curve points, that's where the "Double" come from
- Security is based on the Discrete Log EC Problem ( $\mathbf{P} \cdot \mathbf{k}=\mathbf{Q}$ )


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## Algorithm I

```
% Parameters
- E: }\quad\mp@subsup{y}{}{2}=\mp@subsup{x}{}{3}-3x+0x5a\ldots4b\operatorname{mod}11\ldots5
- n: 1157...4369
- PGE: (ax6b ...96, ax4f\ldots..f5)
- Q Q E: (0xc9...92, 0xb2...46)
```

```
% Operations
    - Seed: S0
    - f(): Si
    -g(): Si.Q (+ more)
    - Out: ri
```

ๆ Keeps an inner state (red) and an outer state (green)

## Algorithm I

## 中 Parameters

- $\mathrm{E}: \quad \mathrm{y}^{2}=\mathrm{x}^{3}-3 x+0 x 5 a . .4$ b mod 11...51
- n : 1157...4369
- $P \in E:(0 x 6 b . .96,0 x 4 f . . . f 5)$
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(T) Inner state is protected by ECDLP

- We cannot, from a ( $Q=k P$ ) point, recover ( $P$ ) and obtain ( $s_{i}$ )

T Seed recovery is protected by ECDLP

- We cannot, from a ( $Q=k P$ ) point, recover $(P)$ and move backwards obtaining ( $s_{i}-1$ )
© Having ( $\mathrm{S}_{j}$ ) means being able to compute ( $\mathrm{S}_{j}>\mathrm{i}$ )
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## Algorithm III

(c) How the Algorithm Really Works:


$$
\mathrm{LSB}_{240}(\mathrm{x}(\mathrm{~s} \cdot \mathrm{P})) \quad \mathrm{s}_{\mathrm{n}+1}
$$

- Notation:
- $r_{i}$ :
- $\operatorname{LSB}_{248}(\ldots)$ : Output the 240 least significant bits
- x(...): Output the $x$ coordinate of a EC point
- input: Optional additional randomness 240 bit random output

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\mathrm{LSB}_{240}(\mathrm{x}(\mathrm{~s} \cdot \mathrm{Q}))
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256 bit inner state
Initial source of randomness

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- $\mathrm{s}_{\mathrm{i}}$ :

256 bit inner state

- seed: Initial source of randomness


## Magic Trick I

Bob, scared of Eve studied the algorithm and found some interesting properties

```
1. With a single ( }\mp@subsup{r}{i}{}\mathrm{ ) all possible 2 2'6}\mathrm{ curve points ( }X,Y)=R= sQ can be bruteforced
    (-) But knowing the outer point R = sQ = (X,Y) point is not useful
    We might now know R = sQ, but we are interested on the s to calculate next states:
        S}=\mp@subsup{L}{SBB}{240}(x(S\cdotP)
    And that means breaking ECDLP (R = sQ)
8 But Bob came with an amazing (and scary) idea
What if Eve knows a secret relation e between P and Q?
```

2. Eve calculates all possible $R=(X, Y)$ from a $r_{i}$. As $(R=s \cdot Q)$ she multiplies it by e
$e \cdot R=e \cdot s \cdot Q$
$e \cdot R=s \cdot e \cdot Q$
$e \cdot R=s \cdot P$

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1. With a single $\left(r_{i}\right)$ all possible $2^{16}$ curve points $(X, Y)=R=s Q$ can be bruteforced
(2) But knowing the outer point $R=s Q=(X, Y)$ point is not useful

We might now know $R=s Q$, but we are interested on the s to calculate next states:

```
s = LSB }\mp@subsup{2}{28}{(x}(x/s\cdotP)
```

And that means breaking ECDLP $(R=s Q)$
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What if Eve knows a secret relation e between F and Q ?
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s = LSBB240 (x (s P P) )
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$$
\begin{aligned}
e \cdot R & =e \cdot s \cdot Q \\
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e \cdot R & =s \cdot P
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## Magic Trick II

© What did just happen?
Eve created backdoored public parameters (P, Q). She fixed P and generated a scalar d:

Then found an e such that $e \cdot d=1 \bmod r$ e.d. $P=e \cdot Q$
$P=e \cdot Q$
With just 240 bits of random output, she can predict all the following bits.
But... $\because \circ$ this was standarized in NIST for 7 years, and used by default in crypto libraries. A WHERE DO THE NIST PARAMETERS CAME FROM?!?!


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d \cdot P=Q
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Then found an e such thate $\cdot d=1 \bmod r$ :

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With just 240 bits of random output, she can predict all the following bits.
But... $\because$ this was standarized in NIST for 7 years, and used by default in crypto libraries.


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"Q [...] could also be generated [...], but NSA kiboshed this idea, and I was not allowed to publicly discuss it, just in case you may think of going there"

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## Backdoor Proof

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8 We have to prove that there is a relation between NIST's P and Q

1. By having $P$ and $Q$ we have to find one of the following numbers:
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Consequences: The Hash of Shame

Wow, people really don't trust their RNGs. The damage done by that NSA Dual EC s**t can still be felt, almost 10 years after the fact.
I have a little bit more faith as I build those. Really not a nation-state mystery to me how they work.

6:09 PM • Apr 29, 2023 • 15.8K Views

## Consequences: The Hash of Shame

```
mjos\dwez
@mjos crypto
If NIST keeps line 2, SHA3-256 hash of the 256-bit random number generated on line 1, I'll just call it "the hash of shame."
It's there because the designers of Kyber think that RNGs (or NIST RBGs) are so bad that they need post-processing like this. You know, just in case.
```

Algorithm 8 Kyber.CCAKEM.Enc( $p k$ )
Input: Public key $p k \in \mathcal{B}^{12 \cdot k \cdot n / 8+32}$
Output: Ciphertext $c \in \mathcal{B}^{d_{u} \cdot k \cdot n / 8+d_{v} \cdot n / 8}$
Output: Shared key $K \in \mathcal{B}^{*}$
1: $m \leftarrow \mathcal{B}^{32}$
2: $m \leftarrow \mathrm{H}(m)$
3: $(\bar{K}, r):=\mathrm{G}(m \| \mathrm{H}(p k))$
4: $c:=$ Kyber.CPAPKE.Enc $(p k, m, r)$
5: $K:=\operatorname{KDF}(\bar{K} \| \mathrm{H}(c))$
6: return $(c, K)$

Question Time -

## Pseudo Random Number Generation

Three Cases Where PRNGs Broke The System
>- PROD v1.3

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$\boldsymbol{g}_{2}$ Simon Lammer
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IIII Graz University of Technology
E Cryptanalysis VO SS23
22nd of June 2023
童 SLIDES \& REPORT

ls.ecomaikgolf.com/slides/randomnumbers/


[^0]:    A The private key d appears in this polynomial's roots

[^1]:    A The private key d appears in this polynomial's roots

